$\langle u_1'(x,t)u_1'(x,\hat{t})\rangle$ = autocorrelation in the near wall region defined by Eq. 19
$\langle u_1'c'\rangle(x_1)$ = turbulent flux of a passive additive
$u_c(x_1)$ = characteristic velocity appearing [illegible] the turbulent model for $\langle u_1'c'\rangle$; $u_c \equiv L($ [illegible] $/T(x_1)$
x = coordinate vector with compone[illegible]s x_1, x_2, x_3
$+$ = superscript denoting a quantity made dimensionless using the wall parameters u^* and ν

Greek Letters

α = defined by Eq. 23
δ_c = characteristic size of the concentration layer near the mass transfer interface (Eq. 7)
ν = kinematic viscosity
τ_H = characteristic relaxation time for the velocity autocorrelation in a frame of reference moving with the average velocity
$\langle \tau_M \rangle$ = mean period between burst
Ω = semi-infinite spatial domain

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Manuscript received July 27, 1981; revision received November 2, and accepted November 5, 1981.

Evaluation of a Stochastic Model of Particle Dispersion in a Turbulent Round Jet

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The accumulation and correlation of data for empirical computations of turbulent particle dispersion is complicated since dispersion is influenced by both particle and turbulence properties. This difficulty can be circumvented by computing dispersion directly, using a stochastic particle dispersion model. The stochastic method requires an estimate of the mean and turbulent properties of the continuous phase. Particle trajectories are then computed using random sampling to determine the instantaneous properties of the continuous phase, similar to a random walk calculation. Mean dispersion properties are obtained by averaging over a statistically significant number of particle trajectories.

Several stochastic particle dispersion models have been proposed. Yuu et al. (1978) use a stochastic dispersion model, which employs empirical correlations of mean and turbulent properties, to analyze their measurements of particle dispersion in jets. Gosman and Ioannides (1981) describe a more comprehensive approach, predicting both flow properties (using a k-ϵ model) and dispersion. The latter procedure is attractive since k-ϵ models yield satisfactory predictions for many of the boundary layer type flows that are encountered with dispersion problems.

The objective of the present investigation was to reexamine the data of Yuu et al. (1978) for particle dispersion in air jets using a stochastic dispersion model similar to Gosman and Ioannides (1981). Shearer et al. (1979) have demonstrated that the present k-ϵ model provides good predictions of existing measurements of mean and turbulent properties within jets. In the following, the present stochastic dispersion model is also calibrated by comparison to predictions by Hinze (1975) and measurements by Snyder and Lumley (1971), for simpler flows, prior to applying the model to the jet dispersion data of Yuu et al. (1978).

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0001-1541-83-6429-0167-$2.00.

THEORY

Continuous Phase

Computations of the properties of the continuous phase were only required for the tests of Yuu et al. (1978). The jet dispersion experiments involve a particle laden round air jet injected into still air which can be modeled as a steady axisymmetric boundary layer flow. Particle mass loadings in the jet were 0.1–0.4% which is sufficiently small so that the particles had a negligible effect on mean and turbulent gas-phase properties. Since nozzle exist Mach numbers were less than 0.3, density variations, kinetic energy and viscous dissipation were neglected with little error. Jet Reynolds numbers were 9,500–48,000 which implies that molecular transport can be ignored in comparison to turbulent transport.

Under these assumptions, the governing equations for the continuous phase, including model transport equations for k and ϵ, are:

$$\frac{\partial u}{\partial x} + \frac{1}{r}\frac{\partial}{\partial r}(rv) = D(u) = D(C) = 0 \quad (1)$$

$$D(k) = \mu_t \left(\frac{\partial u}{\partial r}\right)^2 - \rho\epsilon \quad (2)$$

$$D(\epsilon) = C_{\epsilon 1}\mu_t \left(\frac{\epsilon}{k}\right)\left(\frac{\partial u}{\partial r}\right)^2 - C_{\epsilon 2}\frac{\rho\epsilon}{k} \quad (3)$$

where

$$D(\phi) = \rho u \frac{\partial \phi}{\partial x} + \rho v \frac{\partial \phi}{\partial r} - \frac{1}{r}\frac{\partial}{\partial r}\left(r \frac{\mu_t}{\rho_\phi}\frac{\partial \phi}{\partial r}\right) \quad (4)$$

and all dependent variables denote time-averaged quantities. The specification of the boundary conditions for Eqs. 1–4 and their numerical solution is described by Shearer et al. (1979). Solution of these equations yields u, v, C, k and ϵ throughout the flow field.

A conservation equation for particle concentration appears in Eq. 1, assuming that the particles have the same local mean velocity and turbulent diffusivity as the gas phase. This corresponds to the locally homogeneous flow (LHF) approximation for two-phase flows considered by Shearer et al. (1979). LHF calculations were considered since this approximation is frequently invoked. Furthermore, comparing actual dispersion with the LHF limit provides an indication of the effect of the inertial properties of the particles on particle dispersion.

Particle Motion

Particle trajectories were determined using a Lagrangian formulation of the governing equations. Since $\rho_p/\rho > 200$, for the measurements to be considered, it is reasonable to neglect virtual mass, Bassett forces, Magnus forces, etc. Under these assumptions, conservation of momentum yields

$$\frac{du''_{pi}}{dt} = \left(\frac{3\rho C_D}{4d_p\rho_p}\right)(u''_i - u''_{pi})|\vec{u}'' - \vec{u}''_p| + g_i, \quad i = 1,3 \quad (5)$$

where particle motion is based on the instantaneous velocity of the continuous phase. Gravitational forces were negligible for conditions examined by Hinze (1975) and Yuu et al. (1979), but are included since the effect was appreciable for the experiments of Snyder and Lumley (1971). The position of the particle was determined from

$$\frac{dx''_{pi}}{dt} = u''_{pi}, \quad i = 1,3 \quad (6)$$

Equations 5 and 6 were numerically integrated using a second-order Runge-Kutta algorithm. The trajectories of at least 1,000 particles were computed and averaged to obtain dispersion properties.

Particle Dispersion

Following the method of Gosman and Ioannides (1981) the motion of the particles is tracked as they interact with a succession of turbulent eddies, each of which is assumed to have constant flow properties. Velocity fluctuations were assumed to be isotropic with a Gaussian probability density distribution having a standard deviation of $(2k/3)^{1/2}$. The local distribution is randomly sampled when a particle enters an eddy to obtain the instantaneous velocity as $\vec{u}'' = \vec{u} + \vec{u}'$.

A particle is assumed to interact with an eddy for a time which is the minimum of either the eddy lifetime or the transit time required for the particle to cross the eddy. These times are estimated by assuming that the characteristic size of an eddy is the dissipation length scale, similar to Gosman and Ioannides (1981)

$$L_e = C_\mu^{3/4} k^{3/2}/\epsilon \quad (7)$$

Gosman and Ioannides (1981) compute the eddy lifetime as $t_e = L_e/|\vec{u}'|$, however, we found better agreement with measurements by departing from this particle and employing

$$t_e = L_e/(2k/3)^{1/2} \quad (8)$$

The transit time of a particle was found using the linearized equation of motion for a particle in a uniform flow

$$t_t = -\tau \ln(1 - L_e/(\tau|\vec{u}'' - \vec{u}''_p|)) \quad (9)$$

where $\vec{u}'' - \vec{u}''_p$ is the velocity at the start of the interaction. When $L_e > \tau|\vec{u}'' - \vec{u}''_p|$, the linearized stopping distance of the particle is smaller than the characteristic length scale of the eddy and Eq. 9 has no solution. In this case, the eddy has captured the particle and the interaction time is the eddy lifetime.

CALIBRATION OF MODEL

Since the stochastic model involves some rather arbitrary selections of length and time scales, it was calibrated similar to Gosman and Ioannides (1981) using the fundamental dispersion results of Hinze (1975) and Snyder and Lumley (1971). Assuming constant turbulent diffusivity, Hinze (1975) developed an analytical expression for the diffusion of "marked" fluid particles introduced at a constant rate from a point source into a homogeneous isotropic flow. Comparable results for the stochastic model were obtained by fixing u, k and ϵ. The corresponding turbulent diffusivity of the analytical expression was taken as $C_\mu \rho k^2/\epsilon$, which is consistent with

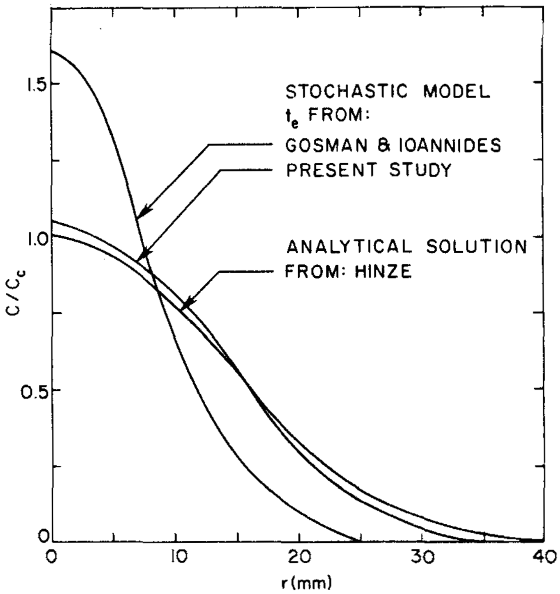

Figure 1. Analytical and stochastic solutions for dispersion of small particles in homogeneous isotropic flow with long diffusion times.

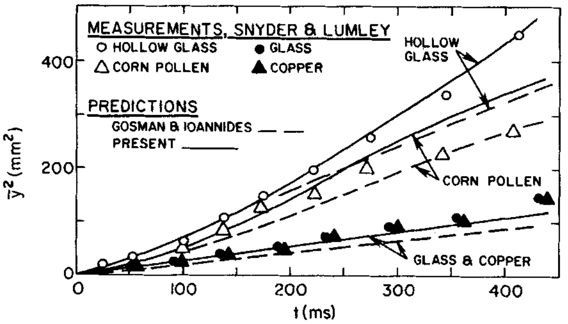

Figure 2. Predicted and measured particle dispersion in a uniform, grid-generated turbulent flow.

the original derivation. The particles were assumed to be small, to correspond with fluid particles. Therefore, the interaction time is always t_e and particle motion is identical to the fluid motion in each eddy.

The radial variation of particle concentration downstream of the point source is illustrated in Figure 1 for the conditions examined by Gosman and Ioannides (1981). Predictions for both stochastic dispersion models are shown, along with the analytical result (C_c was taken from the analytical expression in all cases in order to obtain an absolute comparison). The present stochastic predictions agree favorably with the analytical expression, while the original proposal of Gosman and Ioannides (1981) tends to underestimate the rate of dispersion. (It should be noted that it was not possible to reproduce Gosman and Ioannides (1981) computed results for this flow.)

The second calibration employed the measurements of Snyder and Lumley (1971). These experiments involved the dispersion of individual particles which were isokinetically injected into a uniform turbulent flow downstream of a grid. In this case, interaction times involved both t_e and t_t. The comparison between predictions and measurements is illustrated in Figure 2. The stochastic models provide encouraging agreement with the measurements. The two models yield nearly the same results for heavier particles, where interaction times are mostly fixed by t_t, since this aspect of both models is the same. The present model yields somewhat improved predictions for light particles, where t_e largely controls the interaction time.

PARTICLE LADEN ROUND JET

The experiments of Yuu et al. (1978) involved an air jet containing nearly monodisperse fly ash particles injected into still air. The nozzle was shaped according to the specifications of Smith and Wang (1944) to yield a uniform outlet velocity. Although Yuu et al. (1978) assumed that gas and particle velocities were identical at the nozzle exit, particle trajectory calculations for the stated nozzle shape indicated that this was not the case. Therefore, particle velocities at the nozzle exit were computed to define initial conditions for the present stochastic dispersion calculations. This involved integrating Eq. 5 and 6 assuming that particle and gas velocities were identical at the nozzle inlet. Gas velocities were determined as a function of distance through the nozzle, assuming frictionless flow for the given nozzle shape and neglecting the small contribution of particle drag on the gas flow properties. Computations downstream of the nozzle exit then proceeded as described earlier.

Representative predicted and measured profiles of mean axial gas velocities and particle concentrations are illustrated in Figure 3. The measurements were grouped into ranges of x/d; therefore, predictions are shown for the limits of these ranges. Data scatter has been smoothed by only showing mean values for the measurements.

The k-ϵ flow model yields fair predictions of mean gas velocities, particularly in view of the scatter of the data. Better performance

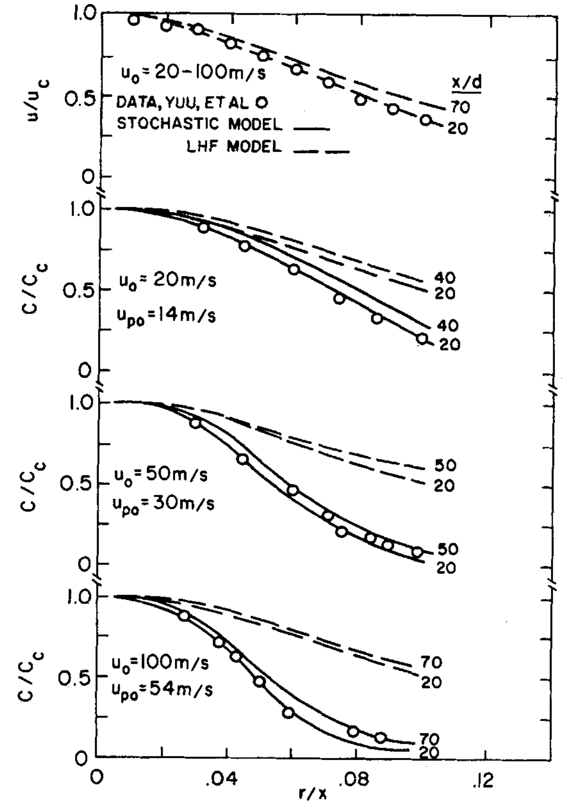

Figure 3. Predicted and measured axial gas velocities and particle concentrations in a dust laden air jet with particle diameter of 20 μm and particle density of 2,000 kg/m³.

of this flow model was generally observed by Shearer et al. (1979) for constant density jets. The stochastic dispersion model, using the computed initial particle velocity, yields quite good agreement with the particle concentration measurements. Similar computations assuming equal particle and gas velocities at the nozzle exit resulted in narrower concentration profiles indicating a significant influence of initial particle conditions on measured dispersion properties. The predictions of the LHF model overestimate the dispersion of particles, therefore, particle inertia is important for these test conditions. The particle trajectory computations also yielded significant differences in particle and gas velocities resulting in maximum particle Reynolds numbers on the order of 50.

CONCLUSIONS

A stochastic model of particle dispersion by turbulence, proposed by Gosman and Ioannides (1981), has been evaluated. The method employs a k-ϵ model to estimate turbulence properties. Dispersion is determined by computing particle motion, with random sampling to obtain instantaneous flow properties, for a statistically significant number of particle trajectories.

The stochastic model yielded good results, particularly when eddy lifetimes were evaluated using Eq. 8. The advantages of the stochastic method are that effects of large relative velocities between the particles and the flow, drag properties at Reynolds numbers greater than the Stokes flow regime, and the variation of local turbulence properties can be readily handled—at least for boundary layer flows.

Additional evaluation of the stochastic model would be desirable. Current prescriptions for eddy length and time scales are somewhat arbitrary and further refinement is needed. The use of a linear drag law to estimate t_t is suspect for the high particle Reynolds numbers often encountered in practice. Finally, while the assumption of isotropic turbulence in the stochastic model is consistent with the

k-ϵ turbulence model, this approximation is questionable for most turbulent flows of interest for dispersion problems.

ACKNOWLEDGMENT

This work was partially supported by NASA Grant No. NAG 3-190, under the technical supervision of R. Tacina of the Lewis Research Center.

NOTATION

C = mean particle concentration
C_D = drag coefficient = $24(1 + Re^{2/3}/6)/Re$, $Re < 1{,}000$; = 0.44, $Re > 1{,}000$
C_i = turbulence model constants: $C_\mu = 0.09$, $C_{\epsilon 1} = 1.44$, $C_{\epsilon 2} = 1.89$
d = nozzle diameter
d_p = particle diameter
g = acceleration of gravity
k = turbulence kinetic energy
L_e = dissipation length scale
r = radial distance
Re = Reynolds number = $\rho|\vec{u}'' - \vec{u}_p''|d_p/\mu$
t = time
t_e = eddy lifetime
t_t = transit time of particle through eddy
u = mean axial gas velocity
u_i, u_{pi} = gas and particle velocity components
v = mean radial gas velocity
x, x_i = axial distance, component of distance
y = vertical distance

Greek Letters

ϵ = dissipation rate of turbulence kinetic energy
μ = viscosity
μ_t = turbulent viscosity = $C_\mu \rho k^2/\epsilon$
ρ = density
σ_ϕ = turbulent Prandtl/Schmidt numbers: $\sigma_{u_i} = \sigma_k = 1$, $\sigma_\epsilon = 1.3$, $\sigma_C = 0.7$
τ = particle relaxation time = $4\rho_p d_p/(3\rho C_D|\vec{u} - \vec{u}_p|)$
ϕ = generic property

Subscripts

c = centerline value
i = component in direction i
p = particle property
o = initial condition

Superscripts

$(\vec{\ })$ = vector quantity
$(\bar{\ })$ = mean quantity
$(')$ = fluctuation with respect to mean quantity
$('')$ = instantaneous value

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Manuscript received September 17, 1981; revision received December 11, and accepted January 13, 1982.